High Performance Dynamic Traffic Assignment Based on Variational Inequality

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Outline

1. Introduction

- Dynamic Traffic Assignment

2. Progress

- Dynamic Network Loading Based on ODE
- Dynamic Network Loading Based on LWR
- Variational Inequality

3. Implementation

- CVODE in SUNDIAL
- Interpolators in boost library
- Ceres Library

Which runs on GPU

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Heat Map Based on Vehicle Density

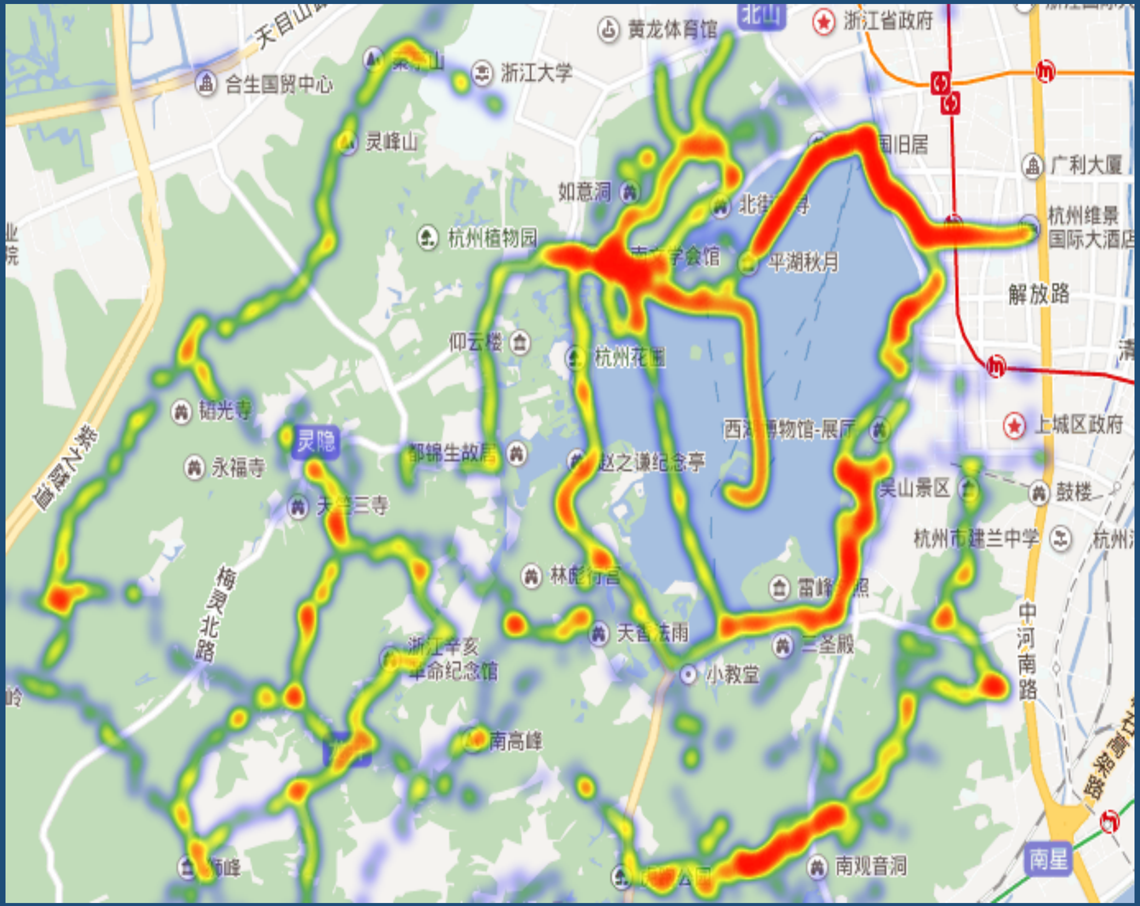
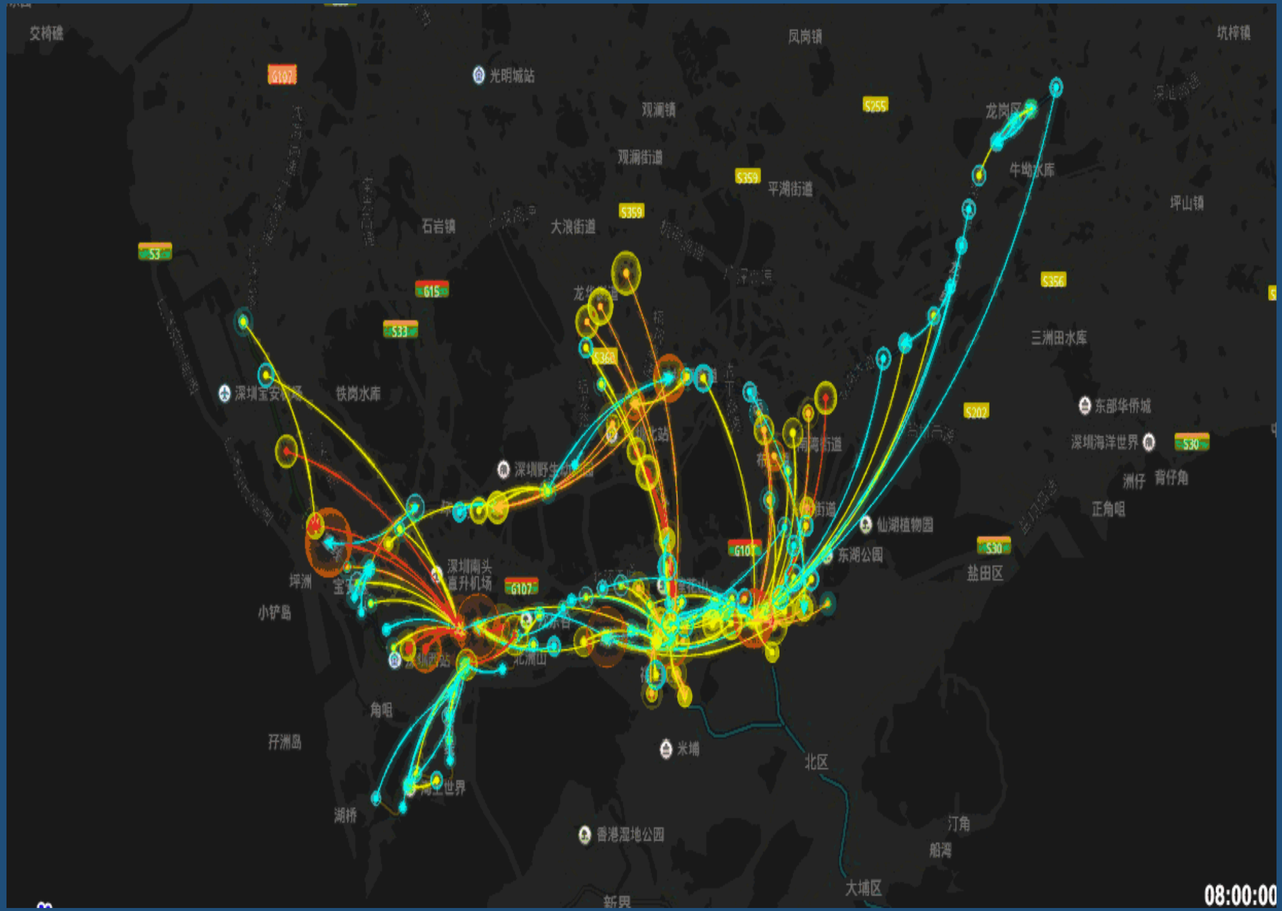
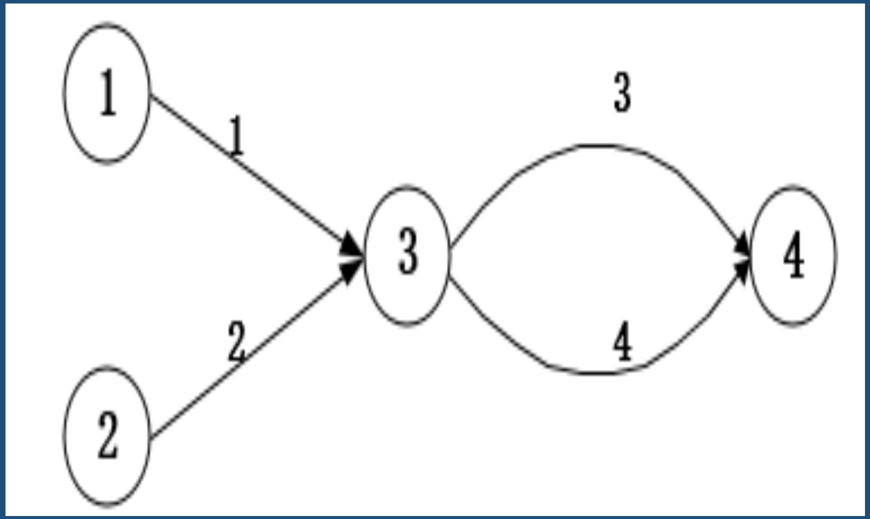


Chart of Traffic Flow



Introduction

Dynamic traffic assignment is the positive modeling of time-varying flows of automobiles on road network consistent with established traffic flow theory and travel demand theory.



A Simple Network

- Nodes:
- Links
- Origin-Destination Pair
- Time cost = Delay = Travel Time

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Introduction

Sioux Fall Network

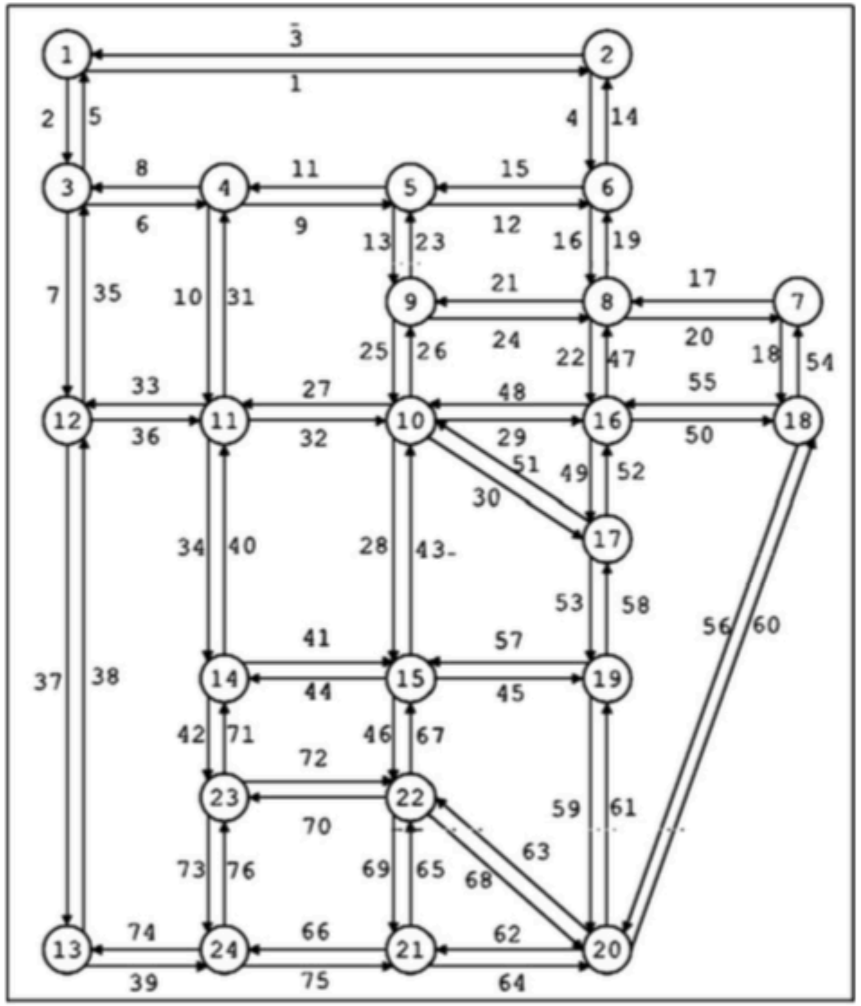
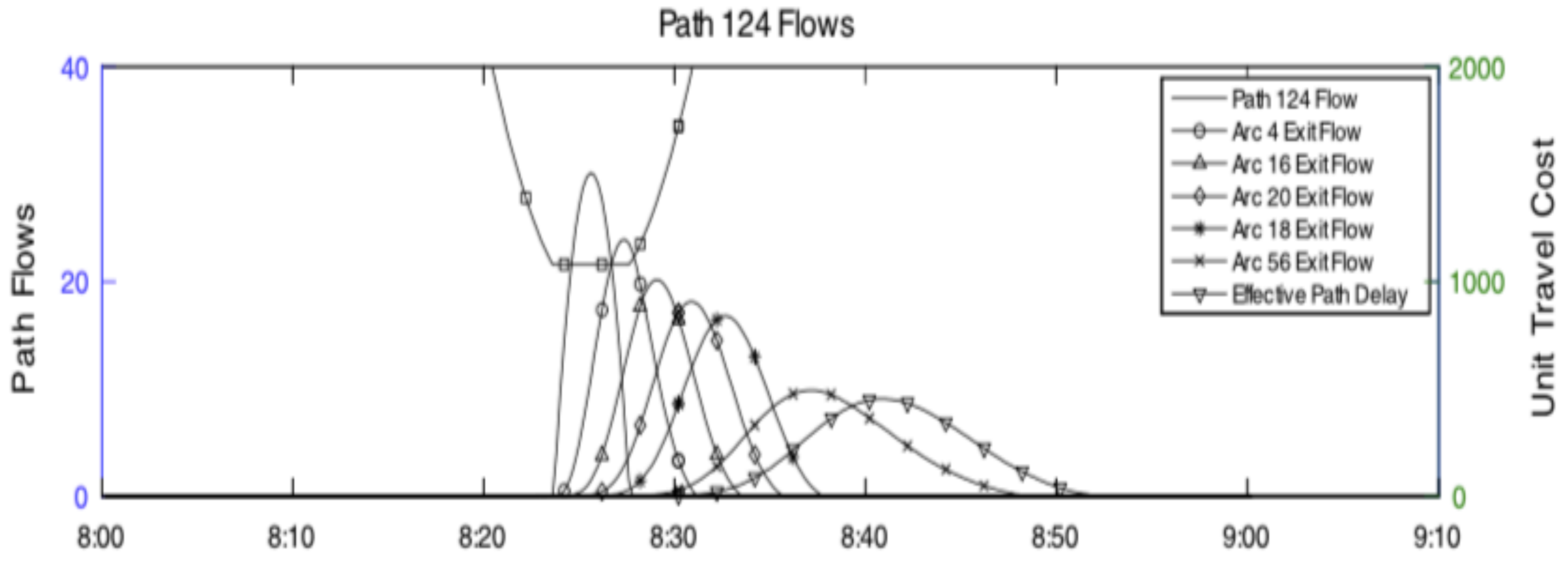


Fig. 1. Sioux Falls network.

Departure rate function and cost function



Introduction

An Optimization Problem

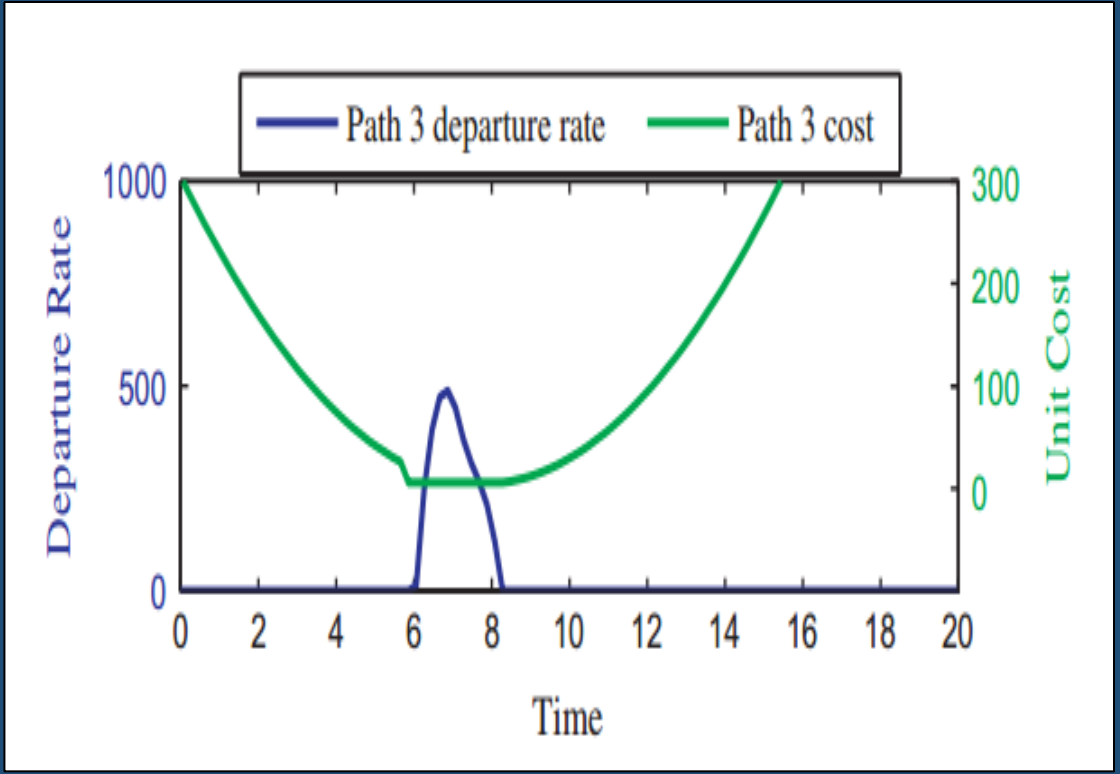
Continuous Time Dynamic User Equilibrium (DUE)

- Users choose the path with the minimum travel time, and the effective travel delay is identical for all the path and departure time of the same travel purpose.

Desired solution:

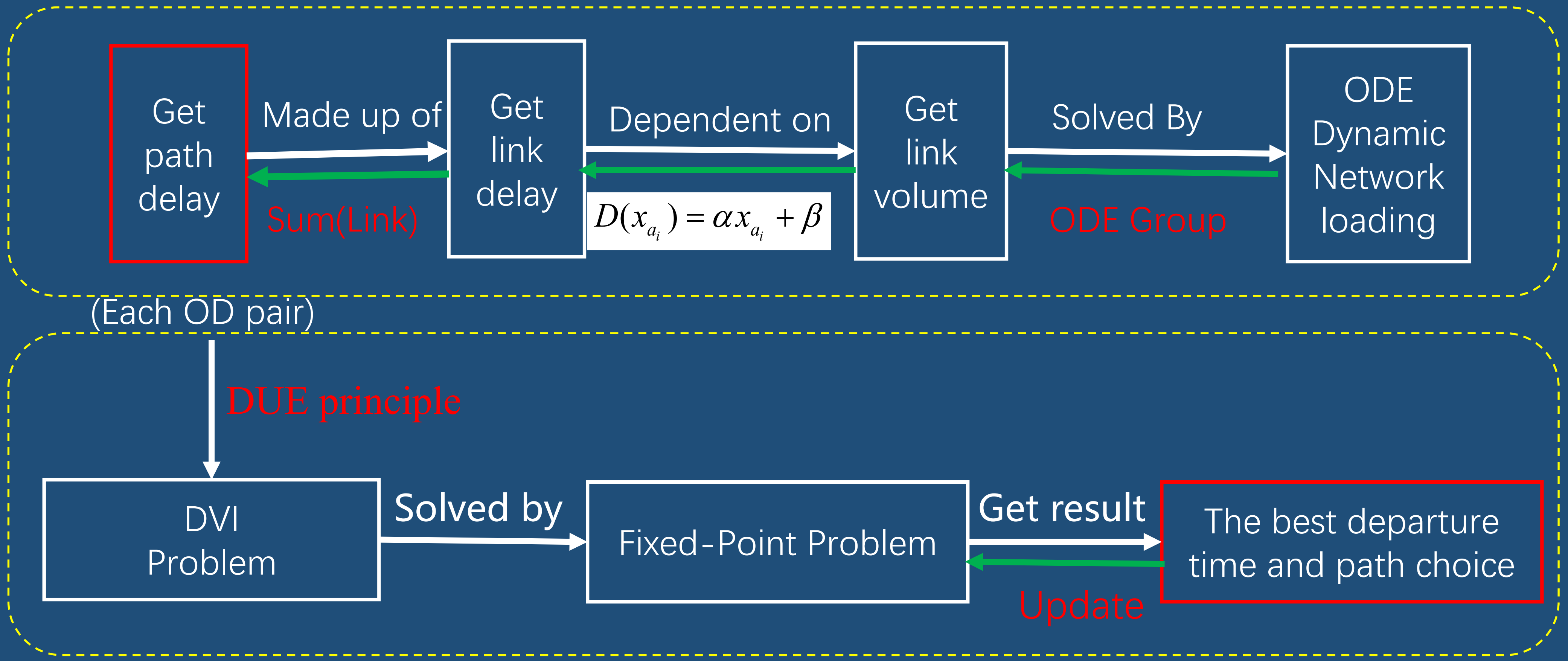
- Desired solution:
  - The departure rate function for each path
  - The corresponding cost function

Fig Departure rates and corresponding travel cost in the DUE solution



Progress

Flow chart



Progress

➢ Part 1: Dynamic Network Loading

Link state equation

$$\frac{dx_{a_1}^p(t)}{dt} = h_p^{\tau,k}(t) - g_{a_1}^p(t) \quad \forall p \in \mathcal{P}$$

$$\frac{dx_{a_i}^p(t)}{dt} = g_{a_{i-1}}^p(t) - g_{a_i}^p(t) \quad \forall p \in \mathcal{P}, i \in [2, m(p)]$$

Medium equation

$$\frac{dg_{a_i}^p(t)}{dt} = r_{a_i}^p(t) \quad \forall p \in \mathcal{P}, i \in [1, m(p)]$$

$$\frac{dr_{a_1}^p(t)}{dt} = R_{a_1}^p(x, g, r, h^{\tau,k}) \quad \forall p \in \mathcal{P}$$

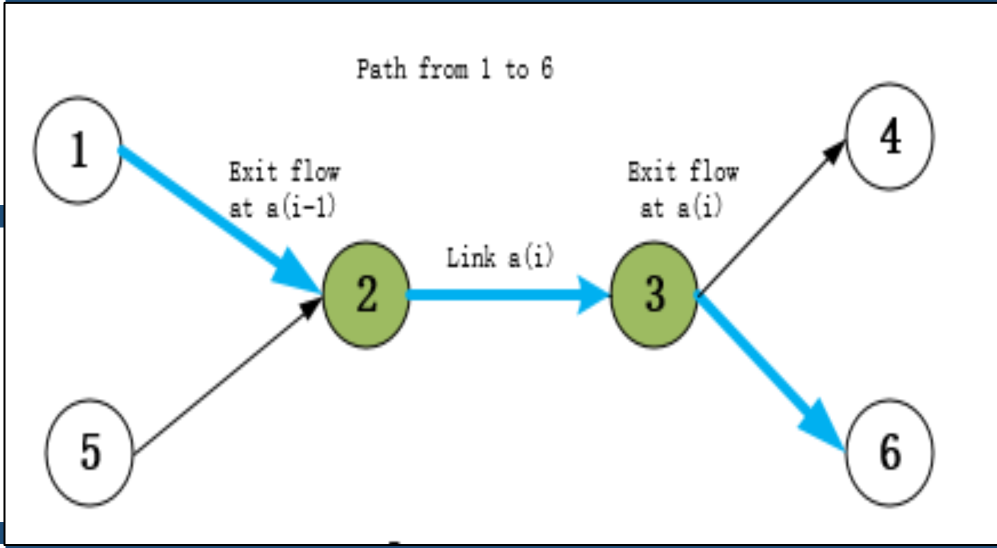
$$\frac{dr_{a_i}^p(t)}{dt} = R_{a_i}^p(x, g, r) \quad \forall p \in \mathcal{P}, i \in [2, m(p)]$$

Initial conditions

$$x_{a_i}^p((\tau - 1) \cdot \Delta) = x_{a_i}^{p,0} \quad \forall p \in \mathcal{P}, i \in [1, m(p)]$$

$$g_{a_i}^p((\tau - 1) \cdot \Delta) = 0 \quad \forall p \in \mathcal{P}, i \in [1, m(p)]$$

$$r_{a_i}^p((\tau - 1) \cdot \Delta) = 0 \quad \forall p \in \mathcal{P}, i \in [1, m(p)]$$

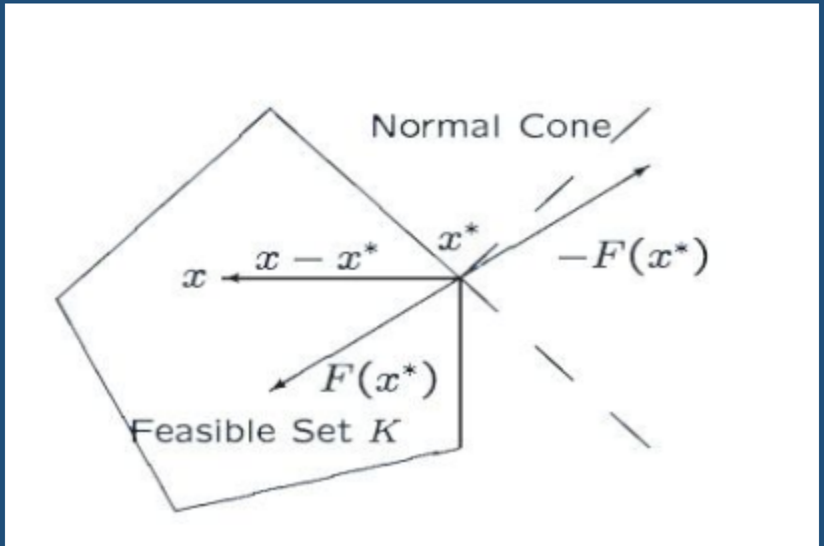


Flow Propagation

Progress

- Part 2: Convert DUE to DVI

$$\left.\begin{array}{l}\text{find } h^* \in \Lambda_0 \text{ such that} \\ \displaystyle\sum_{p \in P_{ij}} \int_{t_0}^{t_f} \Psi_p(t, h^*)\left(h_p - h_p^*\right) dt \geq 0 \quad \forall h \in \Lambda \end{array}\right\}$$



Where spatial condition is

$$\left.\begin{array}{ll} \dfrac{dy_{ij}}{dt} = \displaystyle\sum_{p \in P_{ij}} h_p(t) & \forall (i,j) \in \mathcal{W} \\ y_{ij}(t_0) = 0 & \forall (i,j) \in \mathcal{W} \\ y_{ij}(t_f) = Q_{ij} & \forall (i,j) \in \mathcal{W} \end{array}\right\}$$

satisfy → Flow propagation constraints

Progress

- Part 3: Solve DVI by Fixed-Point Iteration

Equal solution

$$h^* = P_\Lambda[h^* - \alpha\Phi(t, h^*)]$$

For each iteration step

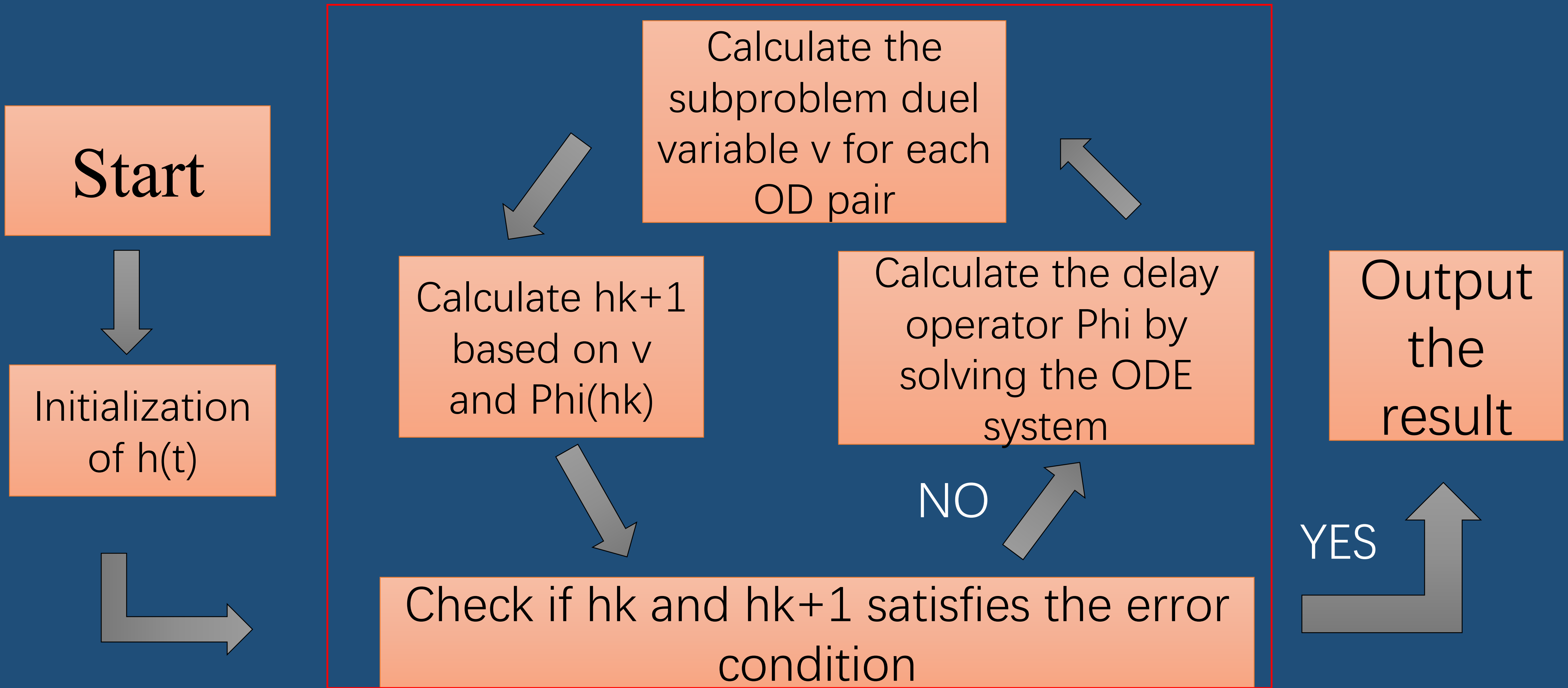
$$\sum_{p\in P_{ij}} \int_{t_0}^{t_f} [h_p^k(t) - \alpha\Phi(t, h_p^k) + v_{ij}]_+ = Q_{ij}$$

Update new departure rate

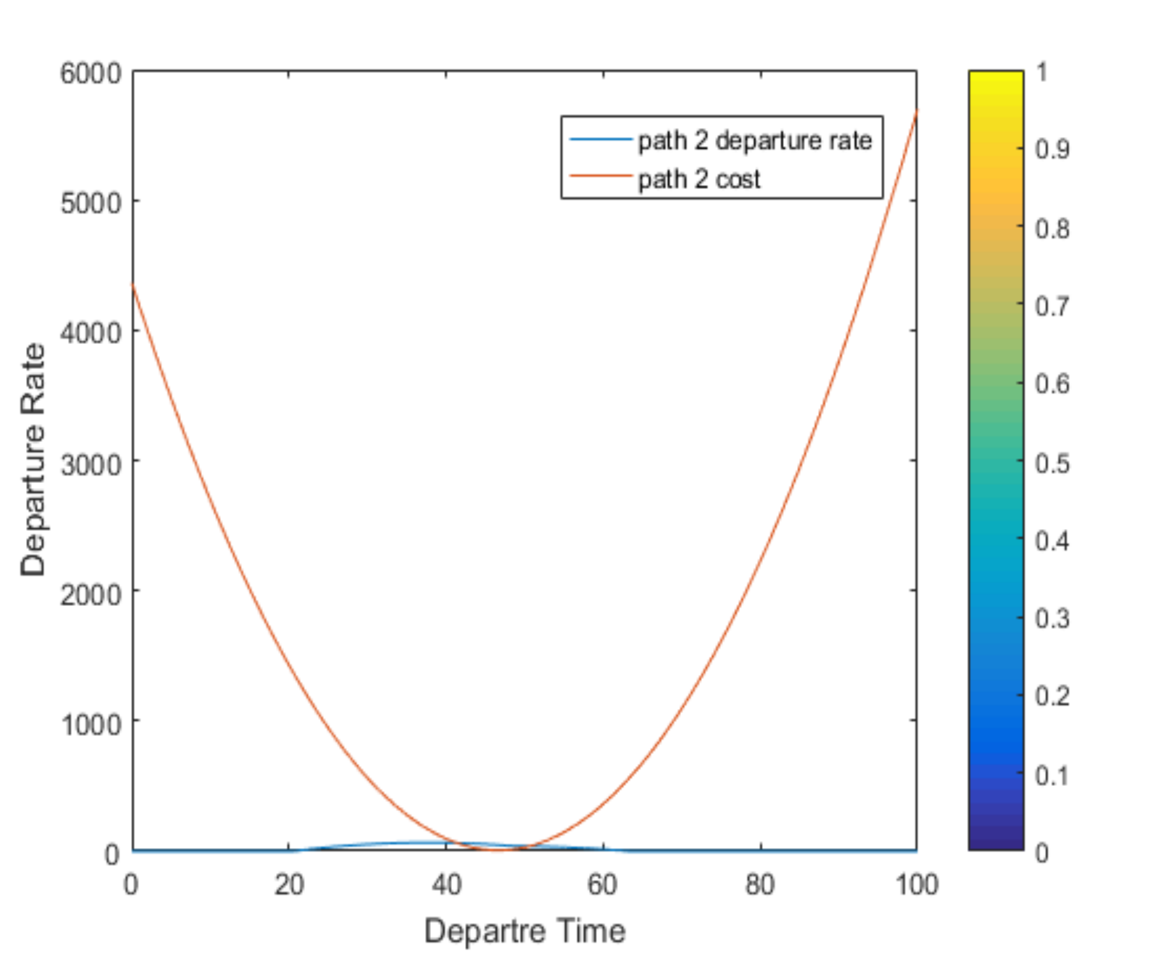
$$h_p^{k+1} = [h_p^k(t) - \alpha\Phi(t, h_p^k) + v_{ij}]_+$$

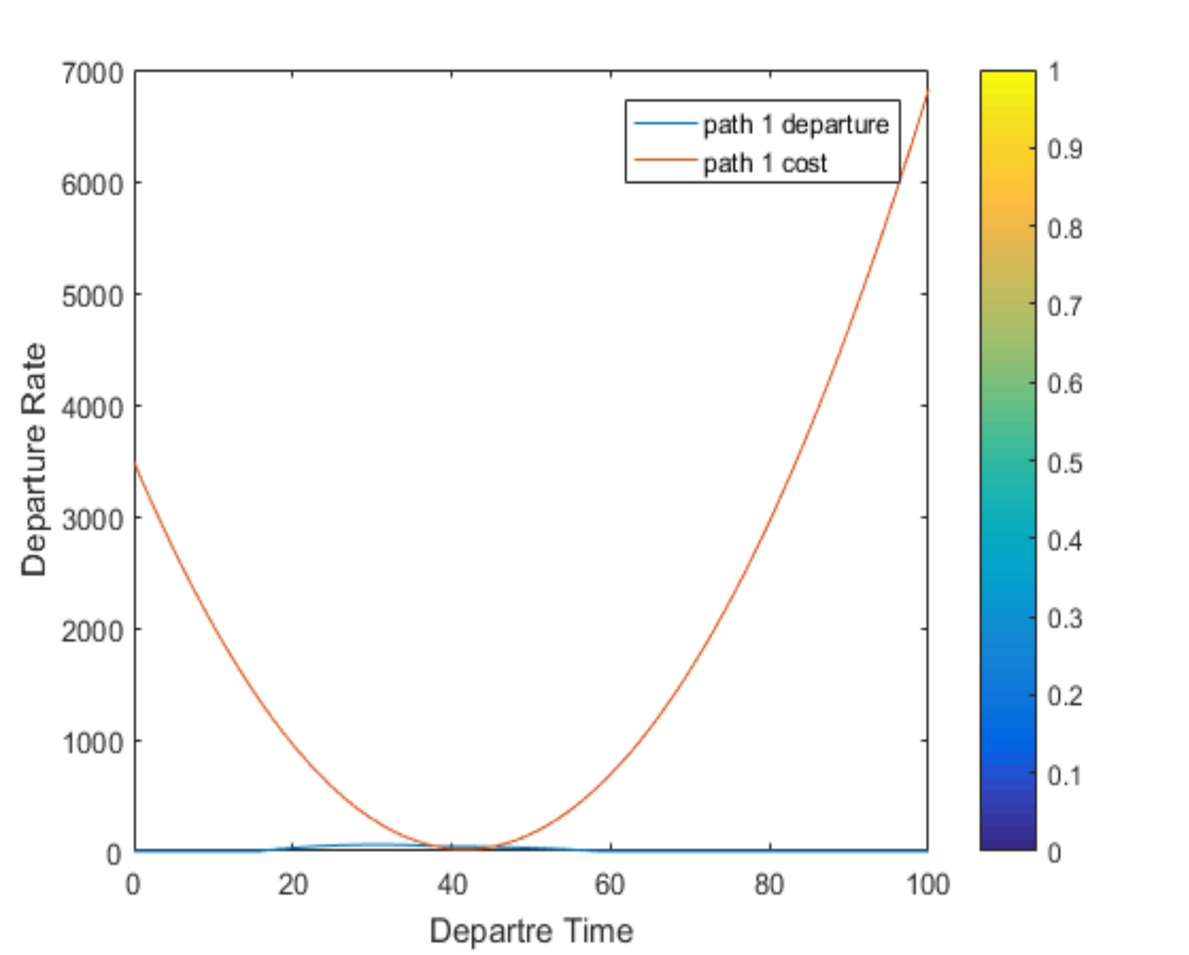
Code structure

Loop of fixed point algorithm



Example


path 2 departure rate
path 2 cost
Departure Rate
Departre Time
6000
5000
4000
3000
2000
1000
0
0
20
40
60
80
100
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0


path 1 departure
path 1 cost
Departure Rate
Departre Time
7000
6000
5000
4000
3000
2000
1000
0
0
20
40
60
80
100
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0

Implementation

Current Work： modify MATLAB
convert MATLAB code to C code
(enlarge graph scale)

ODE solver：
CVODE in SUNDIAL
odeint in boost library

Interpolator:
Interpolators in boost library

Root solver:
CERES Library

Feature Work：Dynamic Network Loading Based on LWR

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Dynamic Network Loading Based on LWR

LWR model

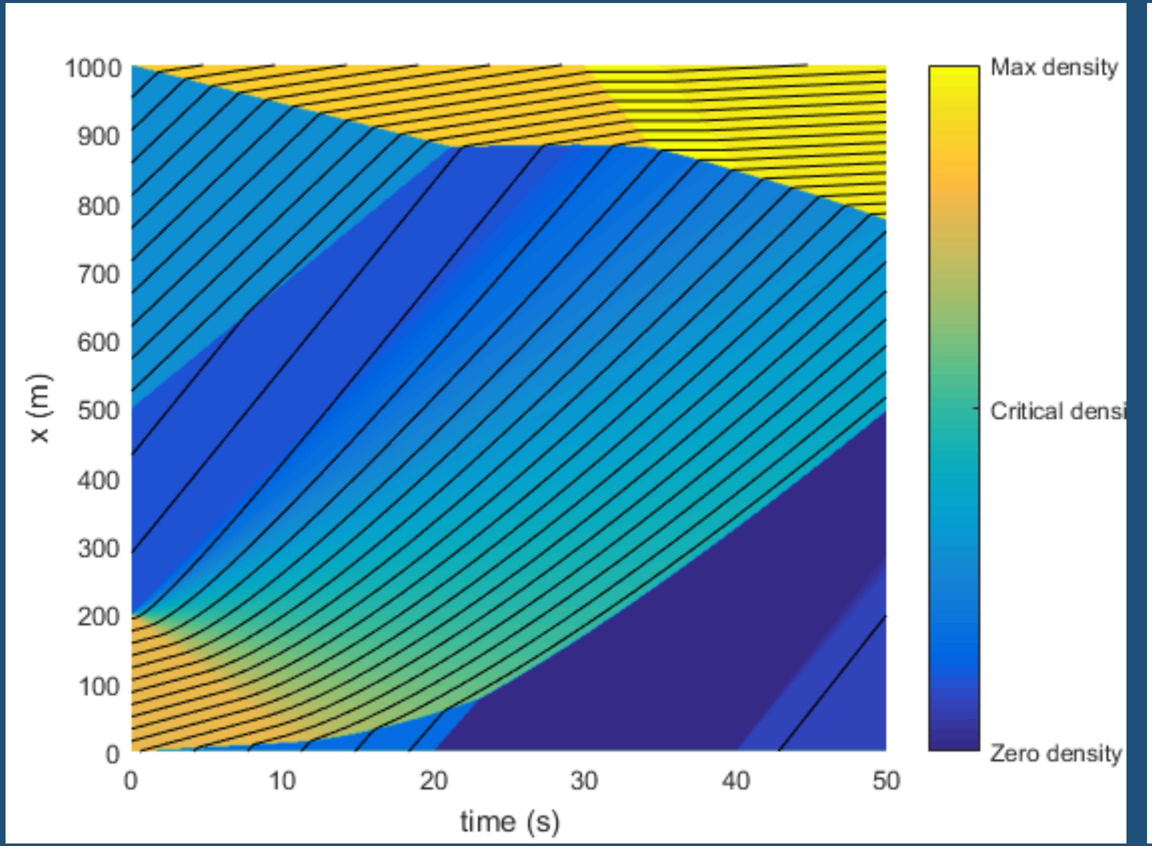
Microcosmic Based on PDE

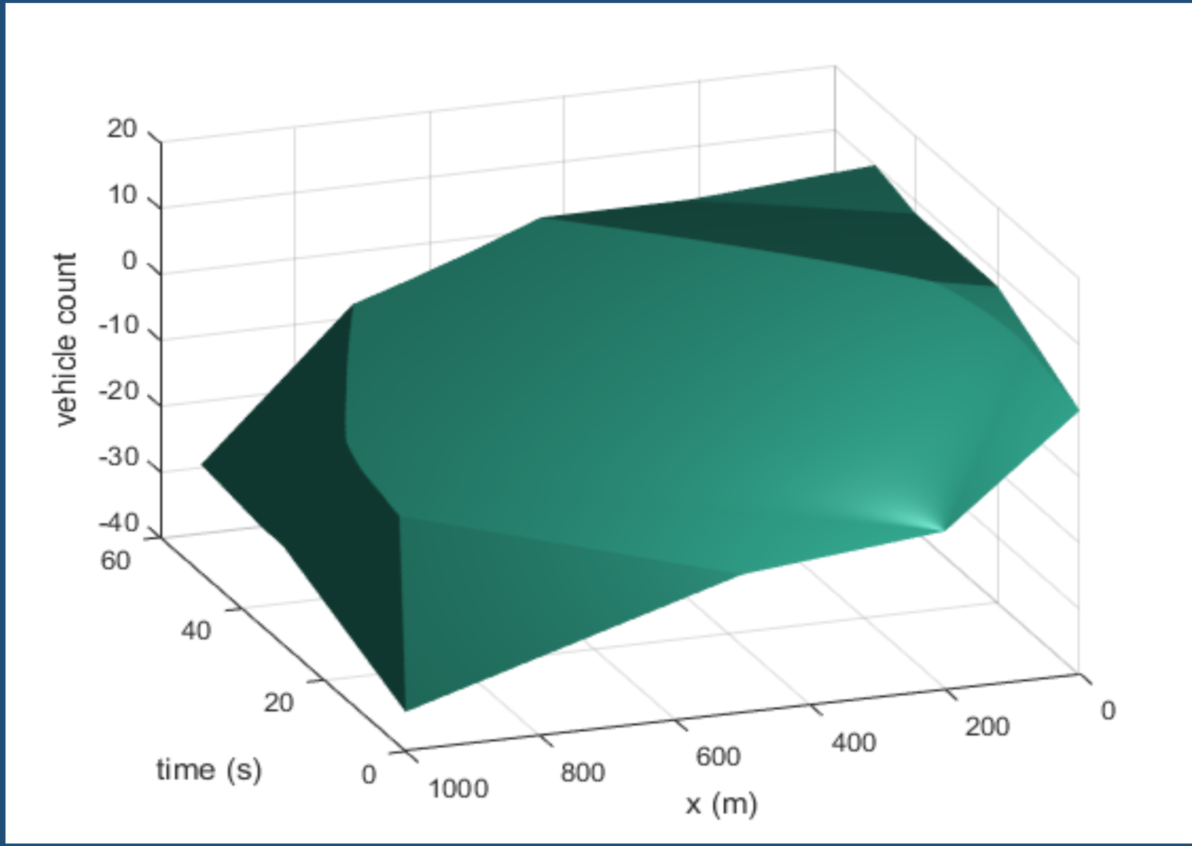
$$\begin{cases} \partial_t \rho^e(t,x) + \partial_x f^e(\rho^e(t,x)) = 0, & (t,x) \in [t_0, t_f] \times [a^e, b^e] \\ \rho^e(0,x) = 0, & x \in [a^e, b^e] \end{cases}$$

Advantages capture

1. Queues and delay
2. Density-speed relationship
3. First-in-First-out principle
4. Route information

Dynamic Network Loading Based on LWR





Link Flow Propagation

Dynamic Network Loading Based on LWR

$$Q^e(t) \doteq \sum_{e\in p} Q^e_p(t), \quad q^e(t) \doteq \sum_{e\in p} q^e_p(t), \quad w^e(t) \doteq \sum_{e\in p} w^e_p(t)$$

$$\frac{d}{dt}Q^e_p(t) = q^e_p(t), \quad \frac{d}{dt}W^e(t) = w^e(t), \quad \forall p \in \mathcal{P}$$

$$q^{e_i}_p(t) = w^{e_{i-1}}_p(t), \quad i \in [1, m(p)], \quad p \in \mathcal{P}$$

$$W^e(t) = \min_{\tau}\left\{Q^e(\tau) + L^e\psi^e\left(\frac{t-\tau}{L^e}\right)\right\}, \quad \forall e \in \mathcal{A}$$

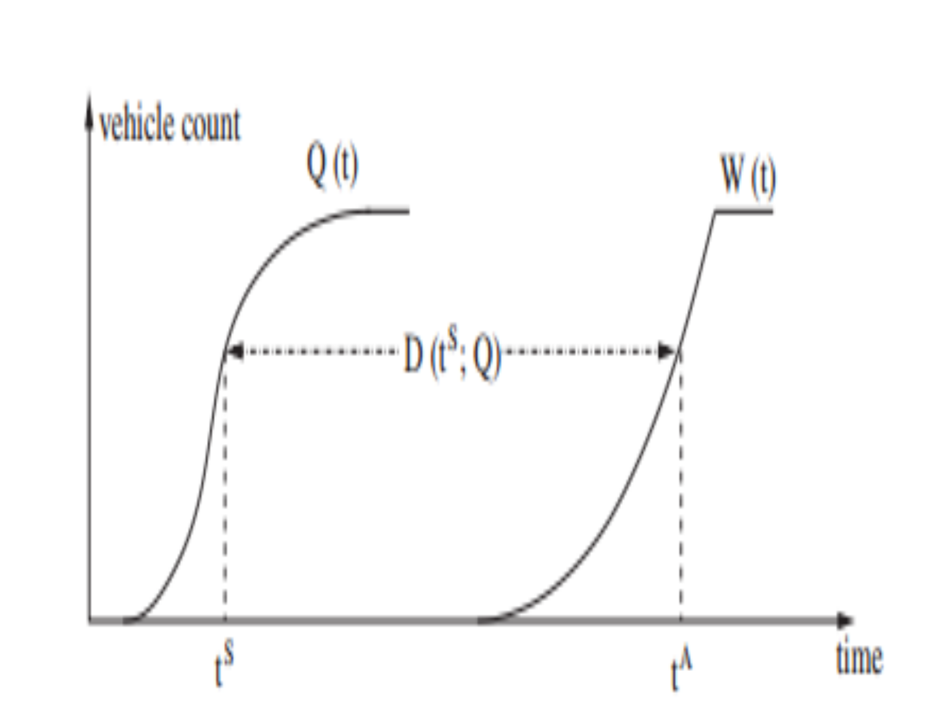
The solution of PDE in the form of Lax-Hopf formula

$$Q^e(t) = W^e(t + D(t;\ Q^e)), \quad \forall e \in \mathcal{A}$$

$$w^{e_i}_p(t + D(t;\ Q^{e_i})) = \frac{q^{e_i}_p(t)}{q^{e_i}(t)}\, w^{e_i}(t + D(t;\ Q^{e_i})), \quad i \in [1, m(p)], \quad p \in \mathcal{P}$$

NetWork Loading

Question



Link delay

Reference

Friesz, T. L., Kim, T., Kwon, C., & Rigdon, M. A. (2011). Approximate network loading and dual-time-scale dynamic user equilibrium. *Transportation Research Part B: Methodological, 45*(1), 176-207. doi:10.1016/j.trb.2010.05.003

Friesz, T. L., Han, K., Neto, P. A., Meimand, A., & Yao, T. (2013). Dynamic user equilibrium based on a hydrodynamic model. *Transportation Research Part B: Methodological, 47*, 102-126. doi:10.1016/j.trb.2012.10.001

End

Thank You