Multi-physics Simulation of the Biomedical Process- Heart Electrophysiology

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Research Plan

- Goal
  - Utilize a set of computer programs to study and simulate the multi-physics phenomena of the heart.

- Steps
  - Study and understand the governing equations for simulating the heart: Monodomain Model and Beeler-Reuter Model.
  - Develop the geometry and mesh of the heart.
  - Program the electrical models: Beeler-Reuter and Monodomain model.
  - Examine the interaction between the electrical and physiological effects of the heart.

Phase 1: Bidomain and Monodomain Model (Tissue Model)

Bidomain Model

$$\nabla \cdot (M_i \nabla v) + \nabla \cdot (M_i \nabla u_e) = \frac{\partial v}{\partial t} + I_{ion}(v, s)$$

After assumptions of homogeneity between the intracellular and extracellular components of the heart the model is simplified into Monodomain Model

$$\frac{\Lambda}{1+\Lambda} \nabla \cdot (M_i \nabla v) = \frac{\partial v}{\partial t} + I_{ion}(v, s)$$

Using the operator splitting schemes, the Monodomain model can be split into a set of equations which can be solved simultaneously at different time steps

$$\frac{\partial v}{\partial t} = -I_{ion}(v, s), \qquad v(t_n) = v^n$$

$$\frac{\partial s}{\partial t} = f(v, s), \qquad s(t_n) = s^n$$

The partial differential equation below can then be solved linearly at different time steps

$$\frac{\partial v}{\partial t} = \frac{\Lambda}{1+\Lambda} \nabla \cdot (M_i \nabla v), \qquad v(t_n) = v_\theta^n$$

Beeler Reuter Model(Cellular Model)

Potential and Current Equations

$$V^{n+1} = V^n + \frac{dV}{dt}\Delta t + O(\Delta t)^2$$

$$\frac{dV}{dt} = -\frac{(i_{k1} + i_{x1} + iNa + iCa - i_{external})}{Cm}$$

$$i_{k1} = \left\{\frac{4\{\exp[0.04(V+85)]-1\}}{\exp[0.08(V+53)] + \exp[0.04(V+53)]} + \frac{0.2(V+23)}{1-\exp[-0.04(V+23)]}\right\}$$

$$i_{x1} = x_1 \cdot 0.8 \cdot \frac{\{\exp[0.04(V+77)]-1\}}{\exp[0.04(V+35)]}$$

$$iNa = (gNa \cdot m^3 \cdot h \cdot j + gNaCa)(V - ENa)$$

Gates Equations (ODE)

$$\frac{dx_1}{dt} = \alpha_{x1}(1-x_1) - \beta_{x1}x_1$$

$$\frac{dm_1}{dt} = \alpha_{m1}(1-m_1) - \beta_{m1}m_1$$

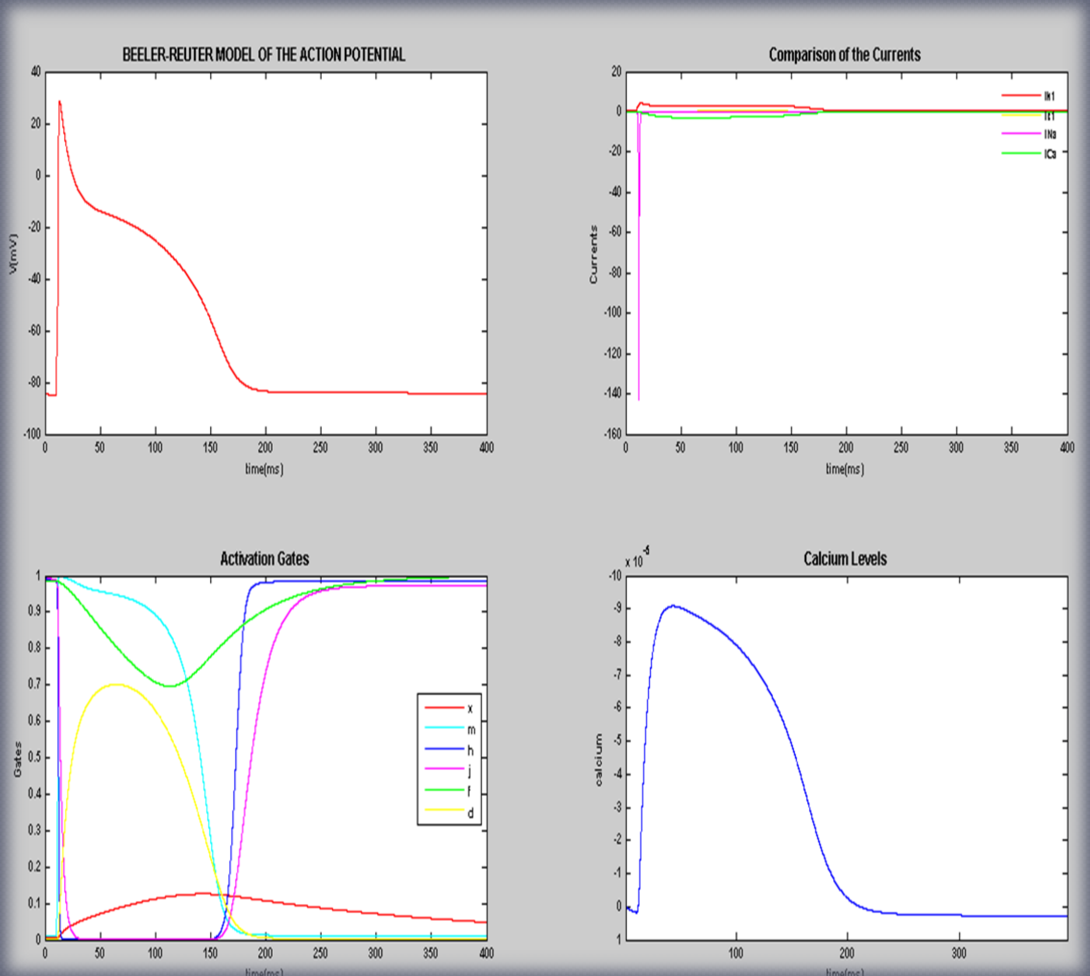
$$\frac{dh_1}{dt} = \alpha_{h1}(1-h_1) - \beta_{h1}h_1$$

$$\frac{dj_1}{dt} = \alpha_{j1}(1-j_1) - \beta_{j1}j_1$$

$$iCa = gCa \cdot d \cdot f \cdot (V - ECa)$$

$$\frac{dd_1}{dt} = \alpha_{d1}(1-d_1) - \beta_{d1}d_1$$

$$\frac{df_1}{dt} = \alpha_{f1}(1-f_1) - \beta_{f1}f_1$$



Phase 2- CUBIT Meshing

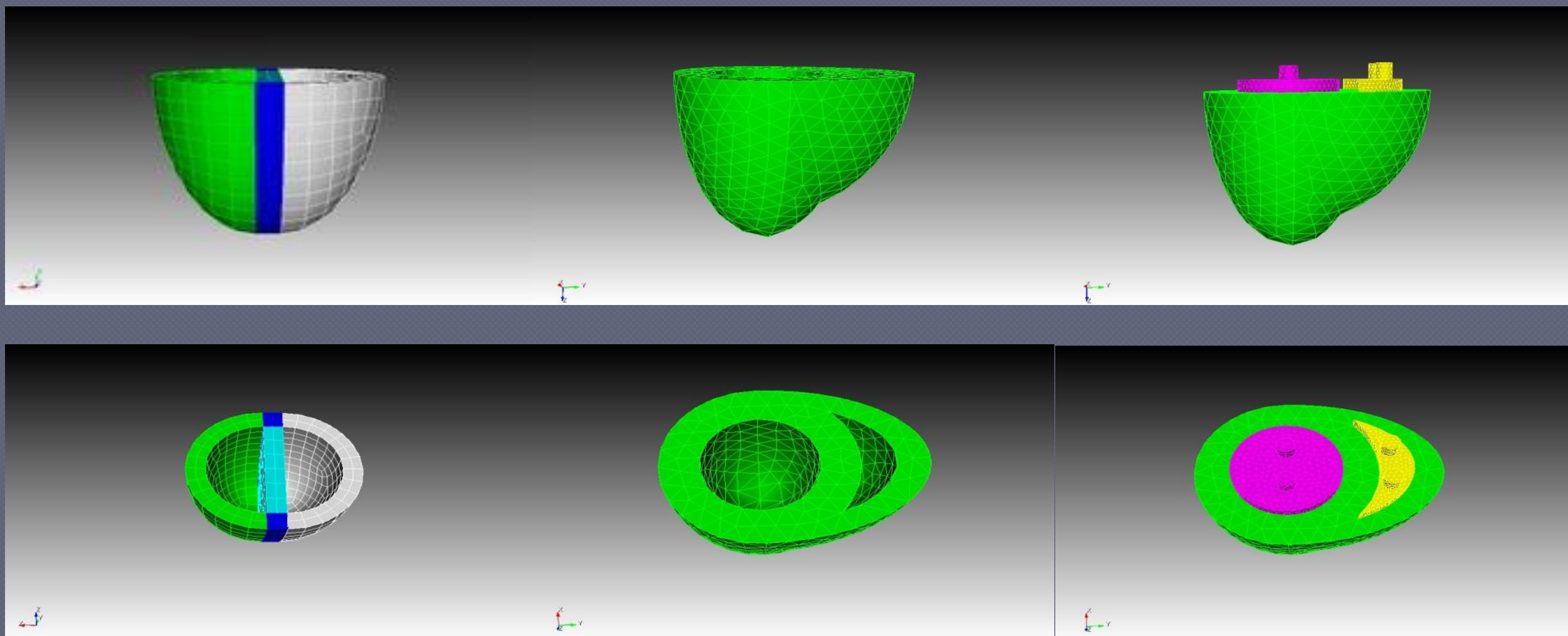


Figure 1 Figure 2 Figure 3

Models of the heart developed and meshed in CUBIT