A MULTI-OBJECTIVE STOCHASTIC PROGRAMMING MODEL FOR DISASTER RELIEF LOGISTICS UNDER UNCERTAINTY

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Introduction

What is disaster relief planning?

- The study of seeking the best plan to prepare for a disaster by allocating the right amount of resources at the right locations before the disaster

A good disaster relief plan:

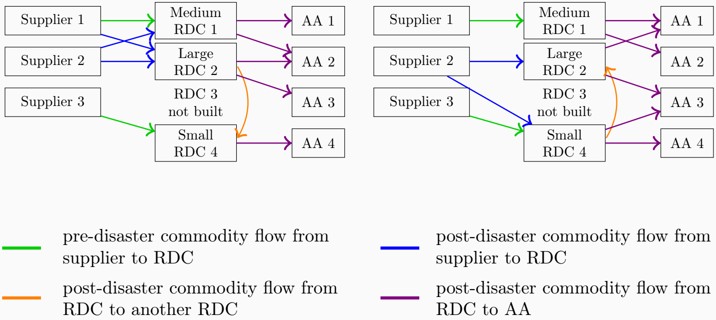
- Meets the demands of affected areas
- Keeps cost low
  - Prevents the surplus of commodities
  - Pre-positions items close to the affected regions, thus reduces transportation costs and increases efficiency
- Applies well to many disaster scenarios

Introduction

- A “multi-objective robust stochastic programming model” by Bozorgi-Amiri et al. (2013)
- Project goals:
  - Improve the aforementioned model in terms of flexibility, speed and solution optimality
  - Apply the new model to real-life cases
  - Implement sensitivity analysis
  - Implement uncertainty quantification (UQ)

Previous Model

- A three-party model: suppliers, candidate relief distribution centers (RDCs) and affected areas (AAs), with four types of commodity flow:



Previous Model

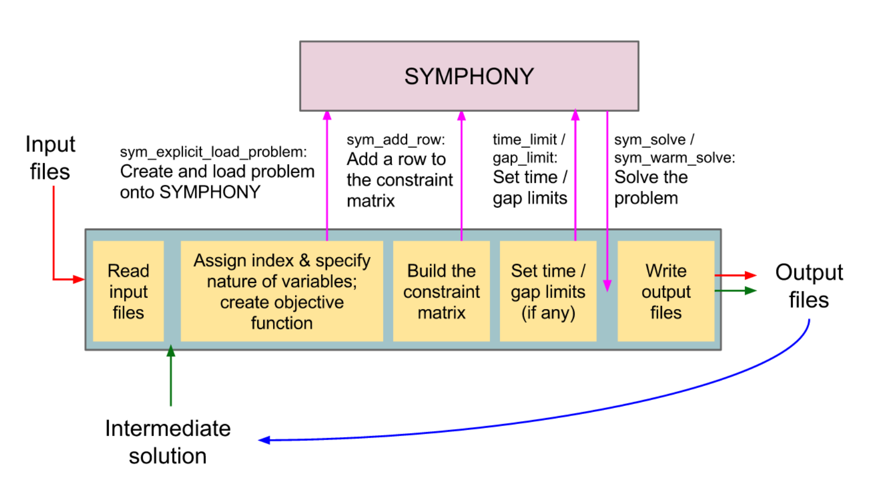
Objectives:

1. To minimize the total costs of preparation and reaction measures
   (a) Preparation measures
      - Setup costs: build relief distribution centers
      - Procuring costs: allocate commodities
      - Transportation costs: deliver commodities
   (b) Reaction measures
      - Procuring costs: allocate more commodities
      - Transportation costs: deliver commodities
2. To maximize affected areas' overall satisfaction by minimizing the sum of maximum shortage at each affected area

System Overview

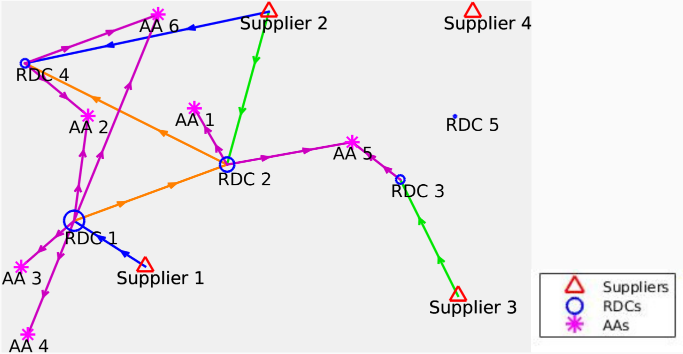
- Use SYMPHONY (an open source mix-integer linear programming solver) as a callable library in C
- Build a C program to read in data, build the objective functions and constraint matrix, and pass to SYMPHONY for the optimal solution
- Features of SYMPHONY used:
  - Time and gap limits
  - Warm start
- star1 (a computer with 12 threads) is used in this research

System Overview



Current Work

- Modifications:
  - Independence of locations of suppliers, RDCs and AAs
  - Mathematical formulation: reducing non-linear constraints to linear constraints



Current model

The parameters include:

- Suppliers (i), RDCs (j) and AAs (k)
- Scenarios (s) and their occurrence probabilities (p)
- Commodities (c) and their required unit space (v), procuring costs (φ), holding costs (h), shortage costs (π) and transportation costs (C)
- Sizes of RDCs (l) and their corresponding setup costs (F) and capacities (Cap)
- Demand at each AA (D) and supply at each supplier (S)
- Fraction of commodity that remains after a disaster at each supplier and RDC (ρ) ($0 \leq \rho \leq 1$)
- Weight assigned to cost variability (λ)
- A very large number (M)

Current model

The variables include:

- Q, X, Y, Z: Amount of commodities delivered under the four types of commodity flow
- I: Amount of inventory at each AA
- b: Amount of shortage at each AA
- δ_{jl}: A binary variable that is 1 if RDC with capacity category l is located at candidate RDC j, and 0 otherwise

Mathematical Formulation

Min. Objective 1

$$= \text{PRE} + \sum_{s \in S} p_s(\text{POST}_s) \quad \text{(expected total cost)}$$

$$+\lambda_1 \sum_{s \in S} p_s \left[\left(\text{POST}_s - \sum_{s' \in S} p_{s'}(\text{POST}_{s'}) \right) + 2\theta_{1s} \right]$$

(cost variance)

Min. Objective 2

$$= \sum_{s \in S} p_s \left(\sum_{c \in C} \max_{k \in K} \{b_{kcs}\} \right) \quad \text{(expected sum of maximum shortage)}$$

$$+\lambda_2 \sum_{s \in S} p_s \left[\left(\sum_{c \in C} \max_{k \in K} \{b_{kcs}\} - \sum_{s' \in S} p_{s'} \sum_{c \in C} \max_{k \in K} \{b_{kcs'}\} \right) + 2\theta_{2s} \right]$$

(maximum shortage variance)

such that

Mathematical Formulation

Amount of commodities delivered to and from each RDC that should balance out in the optimal plan:

$$\sum_{i \in I} X_{ijcs} + \rho_{jcs} \sum_{i \in I} Q_{ijc} + \sum_{j' \in J \setminus \{j\}} Y_{j'jcs} = \sum_{j' \in J \setminus \{j\}} Y_{jj'cs} + \sum_{k \in K} Z_{jkcs}, \ \forall j \in J, c \in C, s \in S$$

(Inward flow of commodity) (Outward flow of commodity)

Amount of commodity at each AA:

$$\sum_{j \in J} Z_{jkcs} - D_{kcs} = I_{kcs} - b_{kcs}, \ \forall k \in K, c \in C, s \in S$$

(Commodity delivered – Demand = Inventory – Shortage)

Mathematical Formulation

Prevent commodity flow to or from a candidate RDC node when an RDC is not built at that node:

$$\sum_{i \in I} \sum_{c \in C} X_{ijcs} \leq M \cdot \sum_{l \in L} \delta_{jl}, \; \forall j \in J, s \in S$$

$$\sum_{j_2 \in J} \sum_{c \in C} Y_{j_1 j_2 cs} \leq M \cdot \sum_{l \in L} \delta_{j_1 l}, \; \forall j_1 \in J, s \in S$$

$$\sum_{j_1 \in J} \sum_{c \in C} Y_{j_1 j_2 cs} \leq M \cdot \sum_{l \in L} \delta_{j_2 l}, \; \forall j_2 \in J, s \in S$$

$$\sum_{k \in K} \sum_{c \in C} Z_{jkcs} \leq M \cdot \sum_{l \in L} \delta_{jl}, \; \forall j \in J, s \in S$$

Ensure at most one type of RDC is built at each candidate location:

$$\sum_{l \in L} \delta_{jl} \leq 1, \; \forall j \in J$$

Mathematical Formulation

Prevent overflow of RDCs:

$$\sum_{i \in I} \sum_{c \in C} v_c \cdot Q_{ijc} \leq \sum_{l \in L} \mathrm{Cap}_l \cdot \delta_{jl}, \forall j \in J$$

Prevent the amount of commodity delivered from each supplier or RDC to exceed the possible amount available:

Before the disaster:

$$\sum_{j \in J} Q_{ijc} \leq S_{ic}, \forall i \in I, c \in C$$

After the disaster:

$$\sum_{j \in J} X_{ijcs} \leq \rho_{ics} \cdot S_{ic}, \forall i \in I, c \in C, s \in S$$

Mathematical Formulation

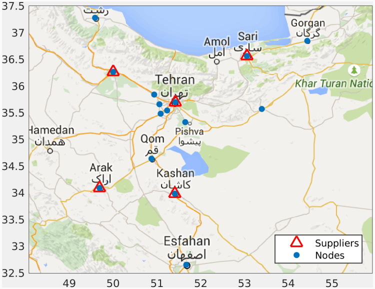
Criteria of θ_{1s} and θ_{2s} for the cost variance measurement:

$$\mathrm{POST}_s - \sum_{s' \in S} p_{s'} (\mathrm{POST}_{s'}) + \theta_{1s} \geq 0, \; \forall s \in S$$

$$\sum_{c \in C} \max_{k \in K} \{b_{kcs}\} - \sum_{s' \in S} p_{s'} \cdot \left(\sum_{c \in C} \max_{k \in K} \{b_{kcs'}\} \right) + \theta_{2s} \geq 0, \; \forall s \in S$$

$$\delta_{jl} \in \{0, 1\}, Q_{ijc}, X_{ijcs}, Y_{j_1 j_2 cs}, Z_{jkcs}, I_{kcs}, b_{kcs}, \theta_{1s}, \theta_{2s} \geq 0$$
$$\forall i \in I, j, j_1, j_2 \in J, k \in K, l \in L, c \in C, s \in S$$

- Case study used in the research of Bozorgi-Amiri et al. (2013)
- 15 nodes for candidate RDCs and AAs, in which 5 are suppliers
- 3 sizes of RDCs: large, medium and small
- 3 types of commodities: water, food and shelter
- 4 scenarios with occurrence probabilities 0.45, 0.3, 0.1, 0.15



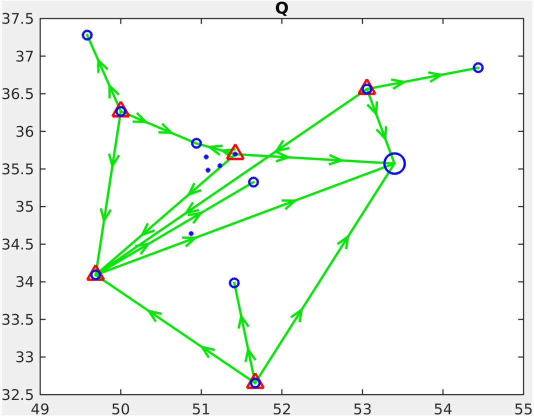
Assumptions:

- Holding costs (h) are assumed to be the current procurement price of commodity (φ)
- Shortage costs (π) are assumed to be ten times the current procurement price of commodity (φ)
- Post-disaster transportation costs are assumed to be 1.8 times of the pre-disaster transportation costs

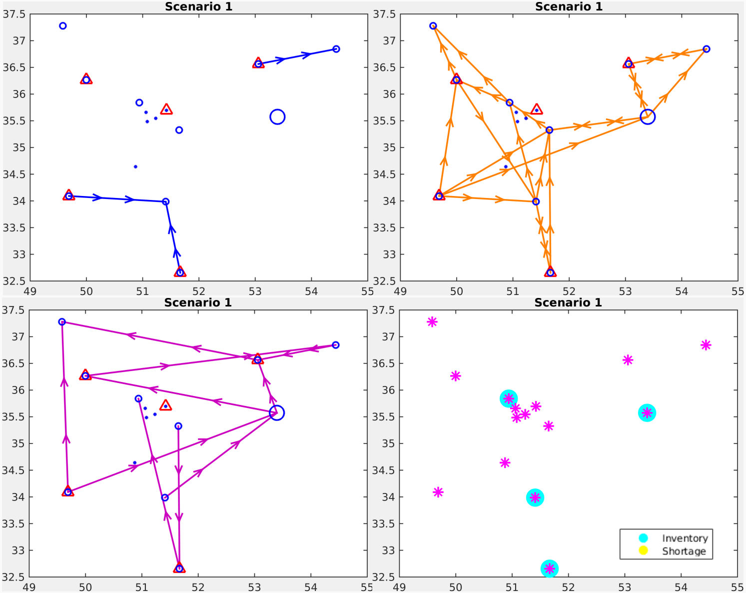
Case Study of Iran: Results

- 1 large RDC and 9 small RDCs should be built
- Minimum objective value: 53022330.6480553597
- Minimum total expected cost: $45.58 million
- Pre-disaster cost: $27.24 million
- Expected post-disaster cost: $18.35 million
- Post-disaster cost in each scenario:
  - Scenario 1: $10.91 million
  - Scenario 2: $20.25 million
  - Scenario 3: $46.11 million
  - Scenario 4: $18.35 million

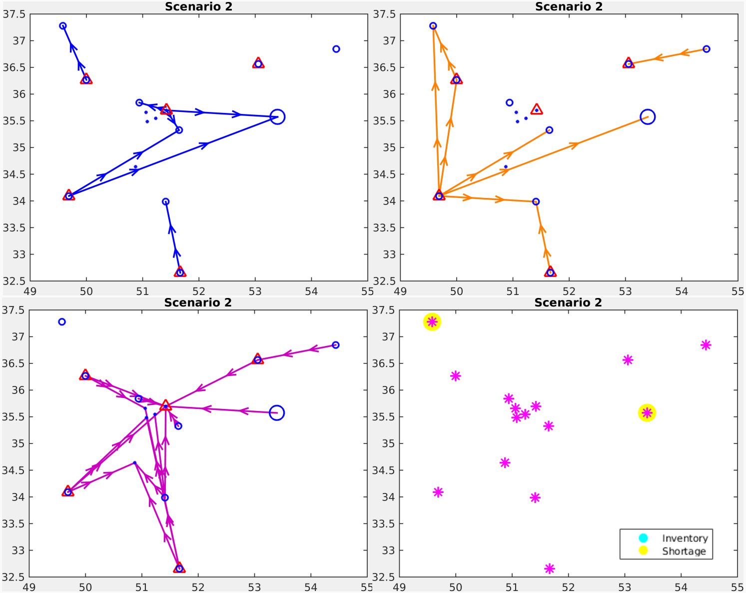
Case Study of Iran: Results



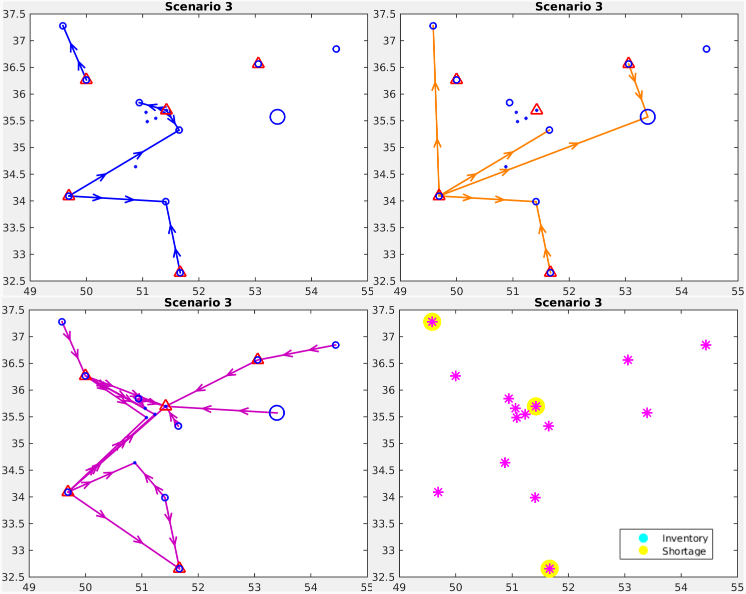
Case Study of Iran: Results



Case Study of Iran: Results



Case Study of Iran: Results



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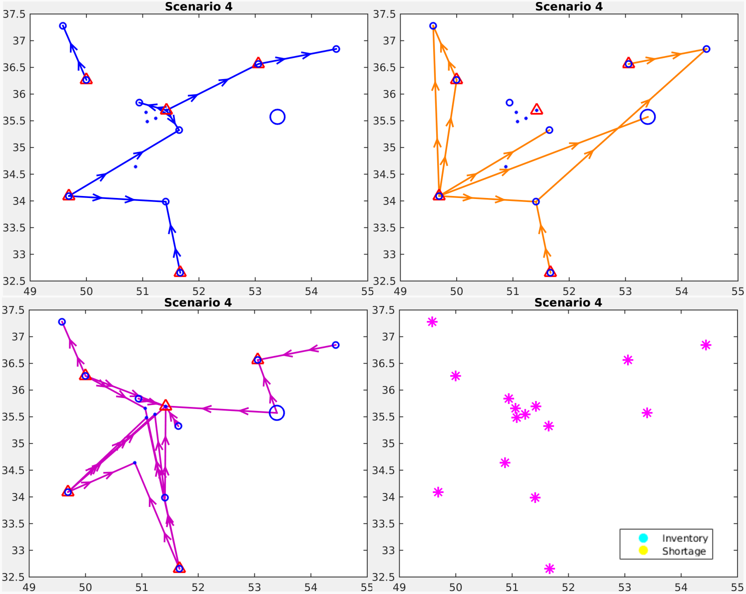

Other cases

- Small case: 8 suppliers, 15 RDCs, 30 AAs, 20 scenarios, 3 sizes of RDCs, 3 types of commodities
  - SYMPHONY takes 224 seconds on star1 to find the optimal solution
- Medium Case: 10 suppliers, 20 RDCs, 80 AAs, 30 scenarios, 3 sizes of RDCs, 3 types of commodities
  - SYMPHONY takes about 107 minutes on star1 to reach an optimality gap of 3% and 13 hours to solve for the optimal solution

Future Work

- Implement parallel computing
  - More efficient solving process
  - Can do multiple cases at a time
- Do real-life cases
  - 500 suppliers, 500 RDCs, 500 AAs and 1000 scenarios
- Sensitivity analysis
- Explore the possibility of using uncertainty quantification in this model
  - PSUADE (Problem Solving environment for Uncertainty Analysis and Design Exploration)

Questions?